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Progress Report

on

Air Force Grant No. AF-AFOSR-76-3075

New Techniques in Numerical Analysis

and Their Application to Aerospace Systems

by

Angelo Miele

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This document summarizes the research done under Air Force Grant No. AF-AFOSR-76-3075 on New Techniques in Numerical Analysis and Their Application to Aerospace Systems, during the period September 1, 1976 to September 30, 1977.

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Progress Report[1]

on

Air Force Grant No. AF-AFOSR-76-3075

New Techniques in Numerical Analysis

and Their Application to Aerospace Systems[2]

by

Angelo Miele[3]

[1]Period September 1, 1976 through September 30, 1977.

[2]This research was supported by the Office of Scientific Research, Office of Aerospace Research, United States Air Force, Grant No. AF-AFOSR-76-3075.

[3]Professor of Astronautics and Mathematical Sciences, Rice University, Houston, Texas.

Contents

I. Introduction

The objective of this investigation is to contribute to numerical techniques of interest in system theory in general and aerospace engineering in particular. Typical areas of matehmatical research are: (a) solution of nonlinear, algebraic or trascendental, equations, (b) solution of nonlinear, ordinary differential equations, (c) solution of linear and nonlinear partial differential equations; (d) mathematical programming problems; and (e) optimal control problems. Typical areas of application are: (i) numerical solution of optimum structural problems, (ii) numerical solution of optimum flight trajectories, and (iii) numerical solution of optimum aerodynamic shapes. Algorithms for digital computer usage are being developed and tested on IBM 370/155.

II. List of Reports Completed

1. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 1, Theory, Rice University, Aero-Astronautics Report No. 126, 1977.

2. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 2, Examples, Rice University, Aero-Astronautics Report No. 127, 1977,

3. MIELE, A., GONZALEZ, S., and WU, A.K., On Testing Algorithms for Mathematical Programming Problems, Rice University, Aero-Astronautics Report No. 134, 1976.

4. MIELE, A., BONARDO, F., and GONZALEZ, S., Modifications and Alternatives to the Cubic Interpolation Process for One-Dimensional Search, Rice University, Aero-Astronautics Report No. 135, 1976.

5. MIELE, A., Some Philosophical Views on Algorithms and Computing Methods in Applied Mathematics, Rice University, Aero-Astronautics Report No. 136, 1976.

III. List of Articles Completed

6. MIELE, A., MOHANTY, B.P., and WU, A.K., Conversion of Optimal Control Problems with Free Initial State into Optimal Control Problems with Fixed Initial State, Journal of the Astronautical Sciences, Vol. 25, No. 1, 1977.

7. MIELE, A., and GONZALEZ, S., On the Comparative Evaluation of Algorithms for Mathematical Programming Problems, Nonlinear Programming 3, Edited by O.L. Mangasarian, R.R. Meyer, and S.M. Robinson, Academic Press, New York, New York, 1978.

8. MIELE, A., Some Philosophical Views on Algorithms and Computing Methods in Applied Mathematics, Proceedings of the Workshop on Decision Information for Tactical Command and Control, Airlie, Virginia, 1976; Edited by R.M. Thrall, C.P. Tsokos, and J.C. Turner; Robert M. Thrall and Associates, Houston, Texas,1976.

9. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, International Journal of Control (to appear).

IV. List of Research in Progress

10. MIELE, A., and WU, A.K., A Transformation Technique for Optimal Control Problems with Linear State Inequality Constraints, Rice University, Internal Memorandum, 1977.

11. MIELE, A., MANGIAVACCHI, A., MOHANTY, B.P., and WU, A.K., Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, Rice University, Internal Memorandum, 1977.

12. MIELE, A., and MOHANTY, B.P., Minimax Problems and the Computation of Chebyshev Optimal Controls, Rice University, Internal Memorandum, 1977.

13. MIELE, A., Transformation Techniques for Optimal Control Problems, Rice University, Internal Memorandum, 1977.

V. Abstracts of Reports Completed

1. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 1, Theory, Rice University, Aero-Astronautics Report No. 126, 1977.

Abstract. A sequential conjugate gradient-restoration algorithm is developed in order to solve optimal control problems involving a functional subject to differential constraints, nondifferential constraints, and terminal constraints. The algorithm is composed of a sequence of cycles, each cycle consisting of two phases, a conjugate gradient phase and a restoration phase.

The conjugate gradient phase involves a single iteration and is designed to decrease the value of the functional while satisfying the constraints to first order. During this iteration, the first variation of the functional is minimized, subject to the linearized constraints. The minimization is performed over the class of variations of the control and the parameter which are equidistant from some constant multiple of the corresponding variations of the previous conjugate gradient phase. For the special case of a quadratic functional subject

to linear constraints, various orthogonality and conjugacy conditions hold.

The restoration phase involves one or more iterations and is designed to restore the constraints to a predetermined accuracy, while the norm of the variations of the control and the parameter is minimized, subject to the linearized constraints. The restoration phase is terminated whenever the norm of the constraint error is less than some predetermined tolerance.

The sequential conjugate gradient-restoration algorithm is characterized by two main properties. First, at the end of each conjugate gradient-restoration cycle, the trajectory satisfies the constraints to a given accuracy; thus, a sequence of feasible suboptimal solutions is produced. Second, the conjugate gradient stepsize and the restoration stepsize can be chosen so that the restoration phase preserves the descent property of the conjugate gradient phase; thus, the value of the functional at the end of any cycle is smaller than the value of the functional at the beginning of that cycle. Of course, restarting the algorithm might be occasionally necessary.

To facilitate numerical integrations, the interval of integration is normalized to unit length. Variable-time

terminal conditions are transformed into fixed-time terminal conditions. Then, the actual time at which the terminal boundary is reached becomes a component of a vector parameter being optimized.

Convergence is attained whenever both the norm of the constraint error and the norm of the error in the optimality conditions are less than some predetermined tolerances. Several numerical examples illustrating the theory in this paper are given in Part 2.

Key Words. Optimal control, gradient methods, conjugate-gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

2. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Part 2, Examples, Rice University, Aero-Astronautics Report No. 127, 1977.

Abstract. In Ref. 1, Cloutier, Mohanty, and Miele developed the sequential conjugate gradient-restoration algorithm for minimizing a functional subject to differential constraints, nondifferential constraints, and terminal constraints. In this report, several numerical examples are presented, some pertaining to a quadratic functional subject to linear constraints and some pertaining to a nonquadratic functional subject to nonlinear constraints. These examples demonstrate the feasibility as well as the rapid convergence characteristics of the sequential conjugate gradient-restoration algorithm.

Key Words. Optimal control, gradient methods, conjugate-gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

3. MIELE, A., GONZALEZ, S., and WU, A.K., On Testing Algorithms for Mathematical Programming Problems, Rice University, Aero-Astronautics Report No. 134, 1976.

Abstract. This paper considers the comparative evaluation of algorithms for mathematical programming problems. It is concerned with the measurement of computational speed and examines critically the concept of equivalent number of function evaluations N_e. Does this quantity constitute a fair way of comparing different algorithms?

The answer to the above question depends strongly on whether or not analytical expressions for the components of the gradient and the elements of the Hessian matrix are available. It also depends on the relative importance of the computational effort associated with algorithmic operations viv-a-vis the computational effort associated with function evalauations.

Both theoretical considerations and extensive numerical examples carried out in conjunction with the Fletcher-Reeves algorithm, the Davidon-Fletcher-Powell algorithm, and the quasilinearization algorithm suggest the following: the N_e concept, while accurate in some cases, has drawbacks in other cases; indeed, it might lead to a distorted view of the relative importance of an algorithm with respect to another.

The above distortion can be corrected through the in-

troduction of a more general parameter $\tilde{N}_e$. This generalized parameter is constructed so as to reflect accurately the computational effort associated with function evaluations and algorithmic operations.

From the analyses performed and the results obtained, it is inferred that, due to the weaknesses of the N_e concept, the use of the $\tilde{N}_e$ concept is advisable. In effect, this is the same as stating that, in spite of its obvious shortcomings, the direct measurement of the CPU time is still the more reliable way of comparing different minimization algorithms.

Key Words. Numerical analysis, numerical methods, computing methods, computing techniques, complexity of computation, philosophy of computation, comparison of algorithms, computational speed, measurement of computational speed, number of function evaluations, equivalent number of function evaluations, unconstrained minimization, mathematical programming.

4. MIELE, A., BONARDO, F., and GONZALEZ, S., Modifications and Alternatives to the Cubic Interpolation Process for One-Dimensional Search, Rice University, Aero-Astronautics Report No. 135, 1976.

Abstract. In this paper, the numerical solution of the problem of minimizing a unimodal function $f(\alpha)$ is considered, where α is scalar. Two modifications of the cubic interpolation process are presented, so as to improve the robustness of the method and force the process to converge in a reasonable number of iterations, even in pathological cases. Modification M1 includes the nonoptional bisection of the interval of interpolation at each iteration of the process. Modification M2 includes the optional bisection of the interval of interpolation: this depends on whether the slopes $f_\alpha(\hat{\alpha}_o)$ and $f_\alpha(\alpha_o)$ at the terminal points $\hat{\alpha}_o$ and α_o of two consecutive iterations have the same sign or opposite sign.

An alternative to the cubic interpolation process is also presented. This is a Lagrange interpolation scheme in which the quadratic approximation to the derivative of the function is considered. The coefficients of the quadratic are determined from the values of the slope at three points: α_1, α_2 , and $\alpha_3 = (\alpha_1 + \alpha_2)/2$, where α_1 and α_2 are the end-points of the interval of interpolation. The proposed alternative is investigated in two versions, Version A1 and Version A2. They differ in the way in which the next interval

of interpolation is chosen; for Version A1, the choice depends on the sign of the slope $f_\alpha(\alpha_o)$; for Version A2, the choice depends on the signs of the slopes $f_\alpha(\alpha_o)$ and $f_\alpha(\alpha_3)$.

Twenty-nine numerical examples are presented. The numerical results show that both modifications of the cubic interpolation process improve the robustness of the process. They also show the promising characteristics of Version A2 of the proposed alternative. Therefore, the one-dimensional search schemes described here have potential interest for those minimization algorithms which depend critically on the precise selection of the stepsize, namely, conjugate gradient methods.

Key Words. One-dimensional search, cubic interpolation process, quadratic interpolation process, Lagrange interpolation scheme, modifications of the cubic interpolation process, alternatives to the cubic interpolation process, bisection process, mathematical programming, interval of interpolation, numerical analysis, numerical methods, computing methods, computing techniques.

5. MIELE, A., Some Philosophical Views on Algorithms and Computing Methods in Applied Mathematics, Rice University, Aero-Astronautics Report No. 136, 1976.

Abstract. This paper summarizes some of the work done by the Aero-Astronautics Group of Rice University in the area of numerical methods and computing methods. It describes some of the philosophical thoughts that have guided this work throughout the years. Recommendations are offered concerning allocation of funds and distribution of funds. Additional recommendations are offered in order to bridge the gap between the top management of government agencies and the academic community.

Key Words. Aerospace engineering, applied mathematics, numerical methods, algorithm research, algorithm development.

VI. Abstracts of Articles Completed

6. MIELE, A., MOHANTY, B.P., and WU, A.K., Conversion of Optimal Control Problems with Free Initial State into Optimal Control Problems with Fixed Initial State, Journal of the Astronautical Sciences, Vol. 25, No. 1, 1977.

Abstract. This note considers optimal control problems involving the minimization of a functional subject to differential constraints, nondifferential constraints, initial conditions, and final conditions. The initial conditions can be partly fixed and partly free. Transformation techniques are suggested, by means of which problems with free initial state are converted into problems with fixed initial state. Thereby, it becomes possible to employ, without change, some of the gradient algorithms already developed for optimal control problems with fixed initial state (for instance, the sequential gradient-restoration algorithm).

The transformations introduced are two: (i) a linear transformation and (ii) a nonlinear transformation. In the linear-quadratic case, the former preserves unchanged the basic structure of the optimization problem, while this is not the case with the latter.

The application of these transformations to a problem of interest in the aerodynamics of a nonslender, axisymmetric body in Newtonian hypersonic flow is shown. It consists of minimizing the pressure drag for given values of the length and the volume, with the nose radius and the base radius being free. After transformations (i) and (ii) are introduced, this problem is solved by means of the sequential ordinary gradient-restoration algorithm (SOGRA) and the sequential conjugate gradient-restoration algorithm (SCGRA).

Key Words. Optimal control, numerical methods, computing methods, gradient-restoration algorithms, problems with free initial state, transformation techniques, sequential gradient-restoration algorithms, applied aerodynamics, Newtonian hypersonic flow, minimum drag bodies.

7. MIELE, A., and GONZALEZ, S., On the Comparative Evaluation of Algorithms for Mathematical Programming Problems, Nonlinear Programming 3, Edited by O.L. Mangasarian, R.R. Meyer, and S.M. Robinson, Academic Press, New York, New York, 1978.

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Both theoretical considerations and extensive numerical examples carried out in conjunction with the Fletcher-Reeves algorithm, the Davidon-Fletcher-Powell algorithm, and the quasilinearization algorithm suggest the following: the N_e concept, while accurate in some cases, has drawbacks in

other cases; indeed, it might lead to a distorted view of the relative importance of an algorithm with respect to another.

The above distortion can be corrected through the introduction of a more general parameter, the time-equivalent number of function evaluations $\tilde{N}_e = T/\tau_0$, where T denotes the CPU time required to solve a particular problem on a particular computer and τ_0 denotes the CPU time required to evaluate the objective function once on that computer. This generalized parameter is constructed so as to reflect accurately the computational effort associated with function evaluations and algorithmic operations.

From the analyses performed and the results obtained, it is inferred that, due to the weaknesses of the N_e concept, the use of the $\tilde{N}_e$ concept is advisable. In effect, this is the same as stating that, in spite of its obvious shortcomings, the direct measurement of the CPU time is still the more reliable way of comparing different minimization algorithms.

Key Words. Numerical analysis, numerical methods, computing methods, computing techniques, complexity of computation, philosophy of computation, comparison of algorithms, computational speed, measurement of computational speed, number of function evaluations, equivalent number of function evalua-

tions, time-equivalent number of function evaluations, unconstrained minimization, mathematical programming.

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Key Words. Aerospace engineering, applied mathematics, numerical methods, algorithm research, algorithm development.

9. CLOUTIER, J.R., MOHANTY, B.P., and MIELE, A., Sequential Conjugate Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, International Journal of Control (to appear).

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The conjugate gradient phase involves a single iteration and is designed to decrease the value of the functional while satisfying the constraints to first order. During this iteration, the first variation of the functional is minimized, subject to the linearized constraints. The minimization is performed over the class of variations of the control and the parameter which are equidistant from some constant multiple of the corresponding variations of the previous conjugate gradient phase. For the special case of a quadratic functional subject to linear constraints, various orthogonality and conjugacy conditions hold.

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The sequential conjugate gradient-restoration algorithm is characterized by two main properties. First, at the end of each conjugate gradient-restoration cycle, the trajectory satisfies the constraints to a given accuracy; thus, a sequence of feasible suboptimal solutions is produced. Second, the conjugate gradient stepsize and the restoration stepsize can be chosen so that the restoration phase preserves the descent property of the conjugate gradient phase; thus, the value of the functional at the end of any cycle is smaller than the value of the functional at the beginning of that cycle. Of course, restarting the algorithm might be occasionally necessary.

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Several numerical examples are presented, some pertaining to a quadratic functional subject to linear constraints and some pertaining to a nonquadratic functional subject to nonlinear constraints. These examples demonstrate the feasibility as well as the rapid convergence characteristics of the sequential conjugate gradient-restoration algorithm.

Key Words. Optimal control, gradient methods, conjugate-gradient methods, numerical methods, computing methods, gradient-restoration algorithms, sequential gradient-restoration algorithms, sequential conjugate gradient-restoration algorithms, nondifferential constraints.

VII. Abstracts of Research in Progress

10. MIELE, A., and WU, A.K., A Transformation Technique for Optimal Control Problems with Linear State Inequality Constraints, Rice University, Internal Memorandum, 1977.

Abstract. This paper considers optimal control problems involving the minimization of a functional subject to differential constraints, terminal constraints, and a state inequality constraint. The state inequality constraint is of a special type, namely, it is linear in some or all of the components of the state vector.

A transformation technique is suggested, by means of which the inequality constrained problem is converted into an equality constrained problem involving differential constraints, terminal constraints, and a control equality constraint. The transformation technique takes advantage of the linearity of the state inequality constraint so as to yield a transformed problem characterized by a new state vector of minimal size. This concept is important computationally, in that the computer time per iteration increases with the square of the dimension of the state vector.

In order to illustrate the advantages of the new transformation technique, several numerical examples are solved by means of the sequential gradient-restoration algorithm for optimal control problems involving nondifferential constraints.

The examples show the substantial savings in computer time for convergence, which are associated with the new transformation technique.

Key Words. Optimal control, numerical methods, computing methods, transformation techniques, sequential gradient-restoration algorithm, nondifferential constraints, state inequality constraints, linear state inequality constraints.

11. MIELE, A., MANGIAVACCHI, A., MOHANTY, B.P., and WU, A.K., Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies, Rice University, Internal Memorandum, 1977.

Abstract. The problem of the axial vibration of a cantilever beam is investigated both analytically and numerically. The mass distribution that minimizes the total mass for a given value of the frequency parameter β is determined using both the sequential gradient-restoration algorithm (SGRA) and the modified quasilinearization algorithm (MQA). Concerning the minimum value of the mass, SGRA leads to a numerical solution precise to 5 significant digits and MQA leads to a numerical solution precise to 7 significant digits.

Comparison of the optimal beam (a variable-section beam) with a reference beam (a constant-section beam) shows that the weight reduction associated with the optimal design depends strongly on the value of the frequency parameter β. This weight reduction is negligible for $\beta \to 0$, is 11.3% for $\beta = 1$, is 78.6% for $\beta = 1.5$, and approaches 100% for $\beta \to \pi/2$.

Key Words. Structural optimization, dynamic optimization, optimal structures, cantilever beams, sequential gradient-restoration algorithm, modified-quasilinearization algorithm, numerical methods, computing methods.

12. MIELE, A., and MOHANTY, B.P., Minimax Problems and the Computation of Chebyshev Optimal Controls, Rice University, Internal Memorandum, 1977.

Abstract. A transformation technique has been developed in order to handle a class of optimal control problems with a Chebyshev minimax performance index within the framework of the classical Mayer problem, wherein the performance index becomes an additional parameter of the problem. The transformation also introduces an inequality constraint, which is akin to a state variable inequality constraint.

Two different numerical optimization schemes have been explored. The first one is an indirect method involving a continuous control and a single subarc. The second one is a direct method involving multiple subarcs. A two-subarc class of algorithms has been developed so as to handle minimax problems having a unique minimax point. Also, a three-subarc class of algorithms has been developed so as to handle minimax problems having a flat minimax solution: here, the middle subarc becomes the flat minimax solution of the problem.

The algorithms developed belong to the class of sequential gradient-restoration algorithms, which is made up of a sequence of two-phase processes or cycles, each consisting of a gradient phase and a restoration phase. The principal pro-

perty of the class of algorithms is that it produces a sequence of feasible suboptimal solutions. To facilitate numerical implementation, the intervals of integration have been normalized to unit length.

To illustrate the transformation technique and the algorithmic studies, several numerical examples are being developed.

Key Words. Optimal control, Chebyshev optimal control, numerical methods, computing methods, minimax problems, transformation techniques, sequential gradient-restoration algorithm, state inequality constraints.

13. MIELE, A., Transformation Techniques for Optimal Control Problems, Rice University, Internal Memorandum, 1977.

Abstract. This paper presents an analysis of the transformation techniques useful in order to bring a wide variety of problems of optimal control within the frame of three standard problems: (i) the minimization of a functional subject to differential constraints and terminal constraints; (ii) the minimization of a functional subject to differential constraints, nondifferential constraints, and terminal constraints; and (iii) the minimization of a functional subject to differential constraints, nondifferential constraints, and multipoint constraints.

The topics treated include: (a) time normalization; (b) problems with free initial state; (c) problems with bounded control; (d) problems with bounded state; and (c) problems with bounded time rate of change of the state.

Key Words. Optimal control, numerical methods, computing methods, gradient-restoration algorithms, transformation techniques, time normalization, free initial state, bounded control, bounded state, bounded time rate of change of the state, nondifferential constraints, multiple subarcs.